The influence of surface radiation on laminar forced-convection film boiling

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A new approach to address the effect of surface radiation on a laminar film boiling flow over a horizontal flat plate is suggested. Previously developed approximate expressions of the thermal boundary-layer thickness and the wall shear stress of a moving surface in a flowing liquid are used to circumvent the complicated simultaneous solution of the vapor and liquid flow. Approximate closed-form expressions to predict the wall heat transfer and skin friction are obtained. For a water–steam system at atmospheric pressure within the wall temperature range considered ($T_w < 800°C$), the surface radiation contribution is negligible in the theoretical wall heat transfer predictions during subcooled $T_\infty = 20°C$ forced-convection film boiling.

Keywords: film boiling; forced convection; radiation

Introduction

In this article, surface-radiation effects are considered in the two-phase boundary-layer analysis of laminar forced-convection film boiling on a horizontal flat plate, and theoretical predictions of wall heat transfer and skin friction are made. This analysis has two objectives. The first objective is to demonstrate the effectiveness of a novel approach to simplifying the forced-convection film boiling analysis if phenomena such as surface radiation are considered. The second, and more important, objective is to estimate the contribution of surface radiation in forced-convection film boiling heat transfer for a water–steam system at atmospheric pressure.

Theoretical analyses of radiation effects on forced-convection film boiling on horizontal flat surfaces[1,3] consider the simultaneous flow of liquid and vapor film and typically use a two-phase boundary layer concept for modeling. Under this concept,[4] the plate surface temperature is assumed to be high enough so that the liquid will form a thin vapor layer, as a result of film boiling, adjacent to the surface after contact (Figure 1). The thin vapor film flow and the liquid flow adjacent to the vapor film are modeled assuming boundary-layer behavior. Governing equations derived for the liquid and vapor phases are coupled at the liquid–vapor interface by the conservation of mass, momentum, and energy. Assuming a nonparticipating vapor medium (for example, valid for a water–steam system at atmospheric pressure),[5] some analyses treat the contribution of surface radiation in film boiling as additive.[6] That is, the analysis first is performed neglecting surface radiation, and a solution is obtained for conduction/convection heat transfer. Later, a radiative heat transfer contribution is added separately to obtain the total heat transfer coefficient. Another approach by Zumbrunnen et al.[1] involved performing the analysis without surface radiation initially. Then, conductive heat transfer coefficients (for saturated film boiling) in the presence of surface radiation are obtained by modifying the conduction heat transfer coefficient (obtained without radiation) proportionally with the surface-radiation heat transfer coefficient. Including surface-radiation effects in the film boiling analysis itself results in a complicated analysis of the governing partial differential equations.[2,3,7]

In the current study, although surface radiation is considered in the film boiling analysis, it is possible to obtain an approximate closed-form solution for wall heat transfer, skin friction, and wall superheat. The simplification occurs because the need for simultaneous solution of the liquid and vapor flow is eliminated by drawing an analogy between the liquid boundary-layer behavior at the liquid–vapor interface and the flow behavior adjacent to the moving surface in a flow liquid. Specifically, previously developed information (the thermal boundary-layer thickness of the liquid and the wall shear stress) of a moving surface in a flowing fluid[8] is used in the current approach to circumvent the simultaneous calculation of vapor and liquid flow, as done in earlier studies.[2,3] The analogy drawn in the current study will be referred to as the *moving wall analogy* hereafter.

Analysis

Consider the forced-convection film boiling flow over a horizontal flat plate maintained at uniform surface temperature (Figure 1). According to Cess and Sparrow,[4] the thin vapor film and the liquid flowing on top of it can be modeled as boundary layers. The current analysis takes as a starting point the two-phase boundary-layer model usually used in the forced-convection film boiling analyses.[1–5,7–11] Following are the general assumptions of the two-phase boundary-layer model.

(1) Steady, two-dimensional, incompressible, and laminar flow is assumed in both phases. Flow velocities in film boiling are generally low enough to justify the incompressibility assumption,[12] and the laminar flow assumption in the vapor film requires $Re_\delta < 100$.[13] Laminar flow limit

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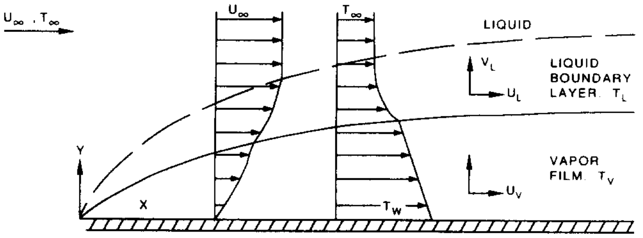

Figure 1 Two-phase boundary-layer forced-convection film boiling flow model

in the liquid flow is currently unknown and would require a stability analysis incorporating the heat transfer, vaporization effect, and the roughness of the vapor film.

(2) Properties of both phases are estimated at their respective film temperatures.[6]

(3) The liquid–vapor interface is assumed to be smooth and at constant saturation temperature (corresponding to the atmospheric pressure). Perhaps the assumption of a smooth liquid–vapor interface is restrictive, and relaxation of this assumption would make the analysis intractable. In practice, the appearance of a smooth liquid–vapor interface has been reported at high liquid subcoolings.[14] As liquid subcooling decreases and the free-stream liquid velocity decreases, the liquid–vapor interface would become wavy and unstable. Transition limit of the liquid–vapor interface from smooth to rough–wavy is currently unknown and may have to be visualized experimentally.

(4) It is assumed that the liquid free-stream velocity is unaffected by the presence of the vapor film. A higher-order theory is necessary to analyze the displacement effect of the vapor film on liquid free-stream velocity.

Additionally, the following assumptions are used in this analysis. The overall effect of these assumptions is discussed later by comparison to numerical solutions and previously published approximate results.

(5) Convective energy transport and inertial effects within the vapor film are assumed to be negligible. The horizontal flat-plate analyses of Cess and Sparrow[4] and Ito and Nishikawa[10] suggest that such an assumption is reasonable as long as $\mathrm{Ja}_v/\mathrm{Pr}_v \ll 12$. For the water–steam system at atmospheric pressure (which is the focus of current study), vapor Prandtl number is about unity, and the current wall temperature range under consideration ($T_w < 1000°\mathrm{C}$) yields vapor Jakob numbers (Ja_v) less than 1. So the above limit is satisfied in this analysis.

(6) The effect of the vertical component of liquid velocity (at the liquid–vapor interface) on the shear stress exerted by the liquid boundary layer on the vapor film and the temperature gradient of the liquid (at the liquid–vapor interface) is assumed to be negligible. Estimates (utilizing the conservation of mass condition at the liquid–vapor interface) of the magnitude of this blowing parameter $(v_L/U_\infty)\sqrt{\mathrm{Re}_x}$ at saturated liquid conditions reveal that it can be as big as 10. So, at saturation liquid conditions, though this assumption is not accurate, it may lead to overestimates of the wall conduction heat transfer. Simple estimates of this parameter a priori at subcooled liquid conditions are difficult. However, as will be shown later, the effect of this assumption along with the other assumptions reveal that this approximation is accurate at subcooled liquid conditions.

(7) The vapor film is assumed to be nonparticipating following Zumbrunnen et al.[1] and Sparrow.[5] This is valid for a water–steam system at atmospheric pressure.

(8) The plate surface and the liquid (including the liquid–vapor interface) is assumed to be opaque and diffuse gray. For

Notation

C_{fx}	Local skin friction coefficient
D	Density ratio (ρ_L/ρ_v)
h_c	Conduction heat transfer coefficient (k_v/δ)
h_{co}	Conduction heat transfer coefficient without radiation (k_v/δ)
h_r	Radiation heat transfer coefficient $\left[h_r = \dfrac{\varepsilon_w \varepsilon_s}{\varepsilon_s + \varepsilon_w - \varepsilon_s \varepsilon_w}\,\sigma\left(\dfrac{T_w^4 - T_{sat}^4}{T_w - T_{sat}}\right)\right]$
h_t	Total radiation heat transfer coefficient $(h_T = h_c + h_r)$
h_x	Local heat transfer coefficient
h_{fg}	Latent heat of vaporization
Ja_L	Jakob number of liquid $[C_{pL}(T_{sat} - T_\infty)/h_{fg}]$
Ja_v	Jakob number of vapor $[C_{pv}(T_w - T_{sat})/h_{fg}]$
k_L	Thermal conductivity of liquid
k_v	Thermal conductivity of vapor
Nu_x	Local Nusselt number
Pr_v	Vapor Prandtl number
Pr_L	Liquid Prandtl number
q''_w	Local wall heat flux
R	Nondimensional density–viscosity product ratio $(\rho_v\mu_v/\rho_L\mu_L)$
RP	Nondimensional surface-radiation contribution parameter $[(h_r x/k_v)/\sqrt{\mathrm{Re}_x}]$
R_1	Nondimensional interfacial velocity parameter $(1 - u_i^*)$
Re_x	Local liquid Reynolds number $(\rho_L U_\infty x/\mu_L)$
Re_δ	Vapor film Reynolds number $(\rho_v u_i \delta/\mu_v)$
T_{sat}	Saturation temperature
T_v	Vapor temperature
T_w	Wall temperature
T_∞	Free-stream liquid temperature
u_v	Vapor streamwise velocity
u_i	Interfacial (liquid–vapor interface) velocity
u_i^*	Nondimensional interfacial velocity (u_i/U_∞)
U_∞	Free-stream liquid velocity
v_L	Vertical component of liquid velocity
y	Normal coordinate

Greek symbols

δ	Vapor film thickness
δ_{tL}	Liquid thermal boundary-layer thickness
ε_w	Emissivity of the plate
ε_s	Emissivity of the liquid–vapor interface
ξ	Boundary-layer thickness ratio (δ/δ_{tL})
μ	Absolute viscosity
ρ	Density
σ	Stefan–Boltzman constant

Subscripts

v	Vapor
L	Liquid
av	Average values

instance, the emissivity of water is nearly one and most of the radiant energy will be absorbed at the liquid–vapor interface.

(9) The radiant energy exchange between the opaque, diffuse-gray wall and the liquid–vapor interface is modeled as occurring between two parallel planes.[1,5] This approximation is made because the vapor film in forced-convection film boiling is generally thin and exhibits a weak dependence on the streamwise coordinate of the plate away from the leading edge.

(10) The liquid–vapor interfacial velocity (u_i) and the ratio of the vapor-layer thickness to the liquid thermal boundary-layer thickness, ξ, are assumed to be weak functions of the streamwise coordinate x and therefore constant. Near the leading edge of the plate, the vapor-film thickness and the liquid thermal boundary-layer thickness may change rapidly; however, they are weak functions of the streamwise coordinate away from the leading edge. Thus, away from the leading edge, it may be reasonable to assume that the parameter ξ is constant. The constant liquid–vapor interfacial velocity assumption is best justified downstream of the leading edge, if the vapor film is thin and does not change rapidly. Such is the case when the liquid is highly subcooled. As the liquid subcooling decreases, the accuracy of this assumption degenerates. As will be shown later, this assumption along with assumption 6 above are reasonable at subcooled liquid conditions.

(11) It is assumed that the ratio of heat transfer coefficients h_r/h_c is constant, independent of the streamwise coordinate x. Under the boundary-layer behavior assumption of the vapor film, that is, $\delta \propto \sqrt{x}$, it is possible to write $d(h_r/h_c)/dx \approx (h_r\delta)/(k_v x)$. Assuming $\varepsilon_w = \varepsilon_s = 1$, $T_w = 600°C$, $T_{sat} = 100°C$, using the properties of a water–steam system at atmospheric pressure considered in this study and taking the reported vapor-film thickness ($\delta \sim 0.25$ mm, $T_\infty = 75°C$) in the literature,[14] the above derivative can be estimated as $0.181/x$. If $d(h_r/h_c)/dx$ is zero, it implies that the ratio (h_r/h_c) is constant and independent of the streamwise coordinate x. For x ranging from 0.1 mm to 150 mm, the derivative $d(h_r/h_c)/dx$ is close to zero for most of the x, except close to the leading edge. Thus, assuming the ratio (h_r/h_c) to be a constant appears reasonable, away from the leading edge for subcooled conditions. The accuracy of this assumption may be reduced as the liquid subcooling decreases.

Under assumptions 1–5 and 7, the governing momentum and energy equations of the steady vapor-film flow, respectively, are

$$\mu_v \frac{\partial^2 u_v}{\partial y^2} \approx 0 \tag{1}$$

$$k_v \frac{\partial^2 T_v}{\partial y^2} \approx 0 \tag{2}$$

The boundary conditions for the vapor film are the no-slip condition and the uniform wall temperature condition at the surface ($y=0$). At the smooth liquid–vapor interface ($y=\delta$), it is assumed that the velocity of vapor is u_i and the temperature is the saturation temperature, T_{sat}. Thus, the boundary conditions are as follows.

At $y=0$, $u_v=0$, $T_v=T_w$

At $y=\delta$, $u_v=u_L=u_i$, $T_v=T_L=T_{sat}$

At $y=\delta$, $\mu_v \frac{\partial u_v}{\partial y} = \mu_L \frac{\partial u_L}{\partial y}$

The motion and energy transport within the liquid viscous layer are governed by the regular boundary-layer equations[4] with the following free-stream boundary conditions.

As $y \to \infty$, $u_L \to U_\infty$, $T_L \to T_\infty$

An energy balance at the liquid–vapor interface ($y=\delta$) yields the following equation:

$$-k_v \frac{\partial T_v}{\partial y} + h_r(T_w - T_{sat}) = -k_L \frac{\partial T_L}{\partial y} + \rho_v h_{fg} \frac{d}{dx}\int_0^\delta u_v\,dy \tag{3}$$

In the above equation, the left-hand side represents the total heat flux arriving at the liquid–vapor interface ($y=\delta$) through the vapor film. The first term on the left side represents the conduction heat flux, and the second term represents the radiation heat flux, where h_r is the radiative heat transfer coefficient. With assumptions 7–9, h_r can be written as

$$h_r = \frac{\varepsilon_w \varepsilon_s}{\varepsilon_s + \varepsilon_w - \varepsilon_s \varepsilon_w} \sigma \left(\frac{T_w^4 - T_{sat}^4}{T_w - T_{sat}} \right) \tag{3a}$$

The first term on the right-hand side of Equation 3 is the heat flux absorbed by the subcooled liquid, and the second term is the heat flux consumed in evaporation.

It is to be noted that the above system of partial differential equations requires a simultaneous solution of the vapor film and the liquid boundary layer either numerically or with an integral procedure. Instead, we propose a new approach, which circumvents the simultaneous solution of the vapor film and the liquid boundary layer. It consists of using previously developed information (the thermal boundary-layer thickness and the wall shear stress) of moving surfaces in flowing fluids[11] to approximate the liquid boundary-layer flow over the liquid–vapor interface. The rest of this article amplifies the details of this approach. Subsequently, the influence of surface radiation is discussed.

Considering the vapor film and integrating Equations 1 and 2 yield the following velocity and temperature profiles within the vapor film:

$$u_v = u_i\left(\frac{y}{\delta}\right) \tag{4}$$

$$\frac{T_w - T_v}{T_w - T_{sat}} = \left(\frac{y}{\delta}\right) \tag{5}$$

It is to be noted that the shear stress of the vapor and liquid boundary layer are yet to be matched. This will be done later on.

Turning our attention to the solution of the interfacial energy balance equation (Equation 3), we note that, in addition to the velocity profile and temperature gradient of the vapor film, we need the temperature gradient of the liquid at the liquid–vapor interface. This gradient can be obtained from the thermal boundary-layer analysis of a moving wall in a flowing fluid. Chappidi and Gunnerson[11] analyzed the heat and momentum transport along moving surfaces in flowing fluids and developed approximate analytical expressions for estimating the local skin friction and heat transfer (and thus the momentum and thermal boundary-layer thickness) of the moving surface as a function of the relative velocity between the solid surface and the moving fluid. These expressions were obtained using a fourth-order polynomial to represent the velocity and temperature profiles in the moving fluid. These expressions are used in the current study to approximate the liquid boundary-layer behavior at the liquid–vapor interface. By using these expressions, assumption 6 is implicitly made. Setting $T_w = T_{sat}$ in the temperature gradient derived for a moving surface in a flowing fluid,[11] the temperature gradient of the liquid at the liquid–vapor interface is written as

$$-k_L\frac{\partial T_L}{\partial y}=2k_L\frac{T_{sat}-T_\infty}{\delta_{tL}} \tag{6}$$

where δ_{tL} is the liquid thermal boundary-layer thickness obtained from the moving wall analysis, which makes use of fourth-order velocity and temperature profiles for the fluid boundary layer. Further, it can be written that the conduction heat transfer coefficient, h_c, is k_v/δ.

Along with assumptions 10 and 11, using Equations 4–6 in the interfacial energy balance equation, Equation 3 can be rewritten as

$$\frac{d\delta}{dx}=\frac{2}{\delta}\left[\frac{\mathrm{Ja}_v R}{\mathrm{Pr}_v u_i^*}\left(\frac{\rho_L}{\rho_v}\right)^2\frac{x}{\mathrm{Re}_x}-\frac{2\mathrm{Ja}_L}{\mathrm{Pr}_L u_i^*}\frac{\xi x}{\mathrm{Re}_x}+\left(\frac{h_r}{h_c}\right)\frac{\mathrm{Ja}_v R}{\mathrm{Pr}_v u_i^*}\frac{x}{\mathrm{Re}_x}\left(\frac{\rho_L}{\rho_v}\right)^2\right] \tag{7}$$

where ξ is the ratio of the vapor-layer thickness to the liquid thermal boundary-layer thickness.

Integrating Equation 7 with the condition $[\delta(x=0)=0]$ yields the following quadratic equation in δ:

$$\delta^2\left[\frac{\mathrm{Re}_x}{x^2}\right]+\delta\left[\frac{8\mathrm{Ja}_L}{\mathrm{Pr}_L u_i^*\delta_{tL}}\left(\frac{\rho_L}{\rho_v}\right)-\frac{4h_r\mathrm{Ja}_v R}{k_v u_i^*\mathrm{Pr}_v}\left(\frac{\rho_L}{\rho_v}\right)^2\right]-\frac{4\mathrm{Ja}_v R}{\mathrm{Pr}_v u_i^*}\left(\frac{\rho_L}{\rho_v}\right)^2=0 \tag{8}$$

Solving the above quadratic equation yields

$$\delta=\frac{-\frac{8\mathrm{Ja}_L}{\mathrm{Pr}_L u_i^*\delta_{tL}}\left(\frac{\rho_L}{\rho_v}\right)+\frac{4h_r\mathrm{Ja}_v R}{k_v u_i^*\mathrm{Pr}_v}\left(\frac{\rho_L}{\rho_v}\right)^2\pm\sqrt{\left[\frac{8\mathrm{Ja}_L}{\mathrm{Pr}_L u_i^*\delta_{tL}}\left(\frac{\rho_L}{\rho_v}\right)-\frac{4h_r\mathrm{Ja}_v R}{k_v u_i^* Pr_v}\left(\frac{\rho_L}{\rho_v}\right)^2\right]^2+\frac{16\mathrm{Re}_x}{x^2}\frac{\mathrm{Ja}_v R}{\mathrm{Pr}_v u_i^*}\left(\frac{\rho_L}{\rho_v}\right)^2}}{\frac{2\mathrm{Re}_x}{x^2}} \tag{9}$$

Consider the positive root and use the following expression developed by Chappidi and Gunnerson[8] for the liquid thermal boundary-layer thickness in Equation 9; that is,

$$\delta_{tL}=\frac{k_1}{\sqrt{\mathrm{Pr}_L}}\frac{x}{\sqrt{\mathrm{Re}_x}} \tag{10}$$

where

$$k_1=2\left(\left(\frac{10}{3}\right)^{1/2}+R_1A_2+R_1^2A_3\right),\qquad A_1=\frac{R_1}{(0.3R_1-0.1824R_1^2)},$$

$$A_2=\left(\frac{5}{6}\right)^{1/2}-\frac{40}{54\sqrt{\mathrm{Pr}_L A_1}},$$

$$A_3=A_2-A_2\sqrt{\frac{640}{243\mathrm{Pr}_L A_1}}-A_2^2\sqrt{\frac{\mathrm{Pr}_L A_1}{400}},\qquad R_1=1-u_i^*$$

Equation 9 results in

$$\frac{\delta}{x}\sqrt{\mathrm{Re}_x}=\frac{\rho_L}{\rho_v}k_2 \tag{11}$$

where

$$K_2=\sqrt{\left(\frac{4\mathrm{Ja}_L}{\sqrt{\mathrm{Pr}_L}k_1u_i^*}-\frac{2\mathrm{Ja}_v RD}{\mathrm{Pr}_v u_i^*}\mathrm{RP}\right)^2+\frac{4\mathrm{Ja}_v R}{\mathrm{Pr}_v u_i^*}}+\frac{2\mathrm{Ja}_v RD}{\mathrm{Pr}_v u_i^*}\mathrm{RP}-\frac{4\mathrm{Ja}_L}{\sqrt{\mathrm{Pr}_L}k_1u_i^*}$$

$$\mathrm{RP}=\frac{\frac{h_r x}{k_v}}{\sqrt{\mathrm{Re}_x}},\qquad D=\frac{\rho_L}{\rho_v}$$

RP is the radiation contribution parameter, and D is the density ratio parameter. Matching the shear stresses of liquid and vapor at the liquid–vapor interface yields

$$\left\langle\mu_v\frac{\partial u_v}{\partial y}\right\rangle_{y=\delta}=\left\langle\mu_L\frac{\partial u_L}{\partial y}\right\rangle_{y=\delta} \tag{12}$$

Using the analytical expression of skin friction derived for moving surfaces in flowing fluids (with a fourth-order velocity profile in the fluid boundary layer)[11] on the right-hand side of Equation 12 results in

$$u_i^*=\frac{\delta}{\mu_v}\rho_L U_\infty\left[\frac{R_1(0.3R_1-0.1824R_1^2)}{\mathrm{Re}_x}\right]^{1/2} \tag{13}$$

Substituting the value of δ from Equation 11 into Equation 13 and rearranging leads to the following form:

$$\mathrm{Ja}_v=\frac{u_i^*}{4}\frac{\mathrm{Pr}_v}{R}\left[\frac{\frac{u_i^{*2}R^2}{R_1(0.3R_1-0.1824R_1^2)}+\frac{8\mathrm{Ja}_L R}{k_1[\mathrm{Pr}_L R_1(0.3R_1-0.1824R_1^2)]^{1/2}}}{1+D^*\mathrm{RP}^*\left\{\frac{u_i^*R}{[R_1(0.3R_1-0.1824R_1^2)]^{1/2}}\right\}}\right] \tag{14}$$

The local skin-friction coefficient is defined as

$$C_{fx}=\frac{\left\langle\mu_v\frac{\partial u_v}{\partial y}\right\rangle_{y=0}}{\frac{1}{2}\rho_L U_\infty^2}=\mu_v\frac{u_i^*}{\frac{1}{2}\rho_L U_\infty\delta} \tag{15}$$

Substituting the vapor-film thickness expression (Equation 11) in Equation 15 and rearranging leads to

$$\frac{C_{fx}\sqrt{\mathrm{Re}_x}}{2}=\frac{Ru_i^*}{k_2} \tag{16}$$

where k_2 is defined in Equation 11. Likewise, defining the local Nusselt number is

$$\mathrm{Nu}_x=\frac{hx}{k_v}=\frac{q_w''x}{(T_w-T_{sat})k_v}=-\frac{\left(-k_v\left\langle\frac{\partial T_v}{\partial y}\right\rangle_{y=0}+h_r(T_w-T_{sat})\right)x}{(T_w-T_{sat})k_v}$$

$$=\frac{x}{\delta}+\frac{h_r x}{k_v} \tag{17}$$

and substituting δ (from Equation 11) results in

$$\frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}}\left(\frac{\mu_v}{\mu_L}\right)=\frac{R}{k_2}+\mathrm{RP}\left(\frac{\mu_v}{\mu_L}\right) \tag{18}$$

Among the parameters appearing in Equations 14, 16, and 18, Pr_v (vapor Prandtl number), Pr_L (liquid Prandtl number), Ja_L (liquid subcooling parameter), D (density ratio parameter), and R (density–viscosity product ratio parameter) are dependent on liquid and vapor properties and assigned (calculating properties at film temperature) on the basis of a water–steam system at atmospheric pressure. The nondimensional interfacial velocity, u_i^* varies from 0 to 1 for a film boiling flow over a horizontal flat plate.[11] Bounds on the radiation parameter, RP, were fixed by considering the no-radiation case (RP=0), and the other limit on RP is taken to be 1. This upper bound on the radiation parameter RP was arbitrarily chosen after doing some trial estimates of RP. For example, in a saturated film boiling flow on a 2-cm plate with a liquid Reynolds number

(Re_x) of 1000 at the trailing edge and assuming $\varepsilon_s = \varepsilon_w = 1$, and $T_w = 600°C$, RP turns out to be about 0.9. By varying the nondimensional interfacial velocity, u_i^* (for a fixed set of Pr_v, Pr_L, Ja_L, D, RP, and R), the wall superheat parameter can be calculated using Equation 14. The corresponding local skin friction and heat transfer then can be evaluated from Equations 16 and 18.

Discussion of results

To evaluate the validity and accuracy of this approach, the results of the current model were compared with the earlier published results of Cess and Sparrow.[4] Additional comparisons with the numerical solution of the full boundary-layer equations of the vapor film and the liquid boundary layer are shown in Figure 2a. The favorable comparisons shown in Figure 2b illustrate and justify the validity of the approximations of 5, 6, and 10 of the current "moving wall" analogy approach in the no-radiation case. More details of this analysis can be found in Chappidi et al.[9]

The surface-radiation parameter, RP, appearing in the current model incorporates the relative importance of surface-radiation effects on the results. This parameterRP$[(h_r x/k_v)/\sqrt{Re_x}]$ is implicitly dependent on other independently prescribable conditions. Under constant free-stream velocity conditions, at a specific streamwise location, with specified ε_s and ε_w, and with constant fluid properties, the parameter RP is dependent on the wall temperature, T_w. The wall superheat parameter, Ja_v, also is related to the wall temperature. Therefore, for every value of Ja_v, a corresponding value of the radiation parameter, RP, exists.

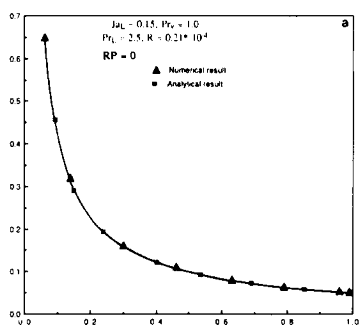

Figure 2(a) Comparison of the results (no-radiation case, RP = 0) of moving wall analogy approach with numerical solution of the complete boundary-layer equations

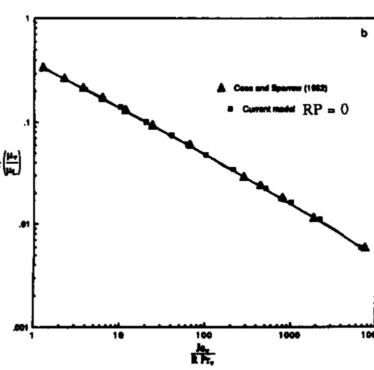

Figure 2(b) Comparison of the results (no-radiation case, RP = 0) of moving wall analogy with Cess and Sparrow[4]

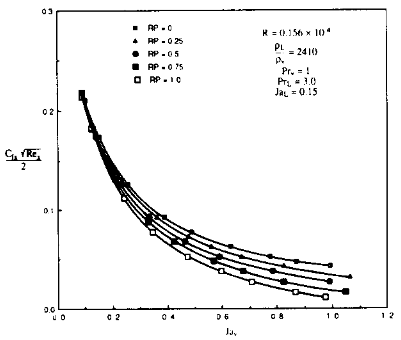

Figure 3 Surface-radiation effect on skin friction under subcooled conditions

In the current analysis, results are presented using two approaches. In the first approach, parametric analysis is done to illustrate the general dependency of the results on the surface radiation. Then, considering some specific flow conditions (prescribing T_w, T_∞, Re_x, ε_s, ε_w, and x) with corresponding nondimensional parameters (Pr_v, Pr_L, Ja_L, D, RP, and R), specific calculations are made to assess the importance of surface radiation.

Figures 3 and 4a illustrate the general influence of the surface-radiation parameter, RP, on the skin-friction parameter and the local wall heat transfer under subcooled conditions ($Ja_L = 0.15$). The chosen parametric values of R, D, and Pr_L are the average values (vapor properties are computed at the film temperature of the vapor and the liquid properties are computed at the film temperature of liquid) within a wall temperature range of $150°C \leq T_w \leq 600°C$ and at subcooled conditions, $T_\infty = 20°C$. RP = 0 refers to the no-radiation case, and RP = 1 refers to the maximum (relative) effect of the surface radiation. Figure 3 shows that the skin friction parameter decreases as the radiation parameter increases from 0 to 1 (implying an increase in surface radiation). This trend is consonant with earlier findings.[2] Our earlier calculations[11] show that the wall shear stress in a film boiling flow decreases with an increase in wall temperature. This occurs because thicker vapor films are formed as the wall temperature increases. Typically, the ratio (u_i/δ) decreases with an increase in wall temperature, and hence the wall shear stress τ_w proportional to (u_i/δ) decreases. Similarly, in the current calculations, an increase in the parameter RP leads to an increase in wall heat transfer (see Equation 18) and hence thicker vapor films are formed, leading to a decrease in the ratio u_i/δ. As a result, the wall shear stress proportional to the ratio u_i/δ decreases with an increase in RP.

Figure 4a shows the effect of the surface radiation parameter, RP, on wall heat transfer calculations under subcooled conditions ($Ja_L = 0.15$). Figure 4a shows that the surface-radiation parameter exerts a negligible influence on local wall heat transfer predictions under highly subcooled conditions ($T_\infty = 20°C$). Although they are indistinguishable in Figure 4a, the numerical values of the local heat transfer parameter, $(Nu_x/\sqrt{Re_x})(\mu_v/\mu_L)$, show a very slight increase as the surface-radiation parameter, RP, increases (see Equation 18). On the whole, Figure 4a shows the lack of effect of surface radiation on local heat transfer predictions in subcooled conditions for a water–steam system at atmospheric pressure.

Effect of the surface-radiation parameter, RP, at an intermediate subcooling level ($Ja_L = 0.075 = > T_\infty \approx 60°C$) is shown in Figures 4b and 4c. Figures 5 and 6 show the influence of the parameter-radiation parameter, RP, on local skin-friction and wall heat transfer parameters under saturated flow conditions ($Ja_L = 0$). As noted earlier for subcooled conditions, the

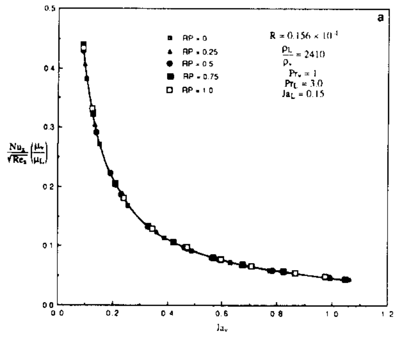

Figure 4(a) Surface-radiation effect on subcooled film boiling heat transfer

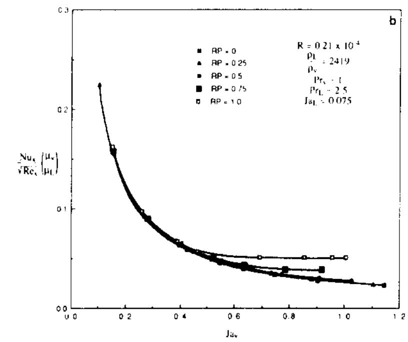

Figure 4(b) Surface-radiation effect on wall heat transfer at intermediate subcooling

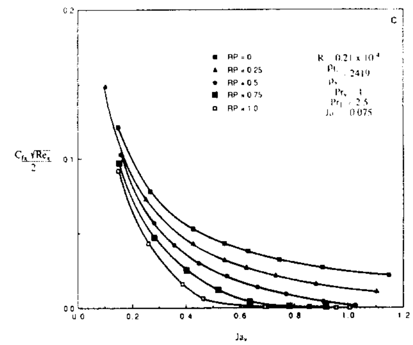

Figure 4(c) Influence of surface radiation on wall skin friction at an intermediate subcooling

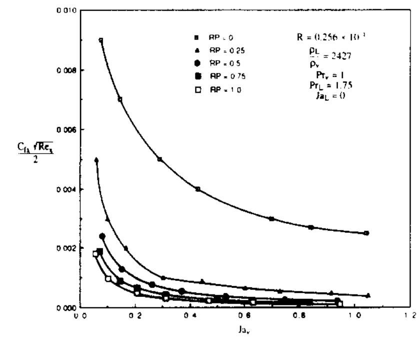

Figure 5 Skin-friction variation with surface-radiation effect under saturation conditions

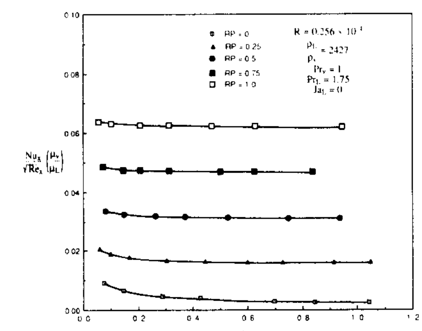

Figure 6 Influence of surface radiation on local Nusselt number for saturation conditions

skin-friction parameter decreases with an increase in the surface-radiation parameter, RP. The increase in the local heat transfer parameter with an increase in the surface-radiation parameter is very evident in Figure 6. For both these trends, the same explanation as given for subcooled conditions holds.

By contrasting Figures 4a and 6, it can be concluded that the surface-radiation effect on wall heat transfer predictions depends greatly on liquid subcooling. Although heat transfer predictions are affected negligibly by surface radiation under subcooled conditions (Figure 4a), they show a strong dependence on surface radiation in saturated flow conditions (Figure 6). The wall heat flux in a forced-convection film boiling flow can be written as

$$q''_w = -\left\langle k_v \frac{\partial T_v}{\partial y} \right\rangle_{y=0} + h_r(T_w - T_{sat}) \tag{19}$$

where the first term on the right side is approximately $(T_w - T_{sat})k_v/\delta$. For subcooled conditions, the vapor film is thin, and as a result, the wall conduction heat flux is promoted. Under saturated flow conditions ($Ja_L = 0$), the vapor film is thicker because all the heat flux arriving at the liquid–vapor interface is used in the production of vapor. Thicker vapor films lead to a deterioration in the wall conduction heat flux. As a result, it can be conjectured from Equation 19 that the relative importance of radiative heat transfer is more pronounced in saturation conditions.

As stated earlier, the parameter RP depends on the wall temperature (T_w), the streamwise position (x), the local Reynolds number (Re_x), the vapor thermal conductivity (k_v), the emissivity of the surface (e_w), and the emissivity of the liquid–vapor interface (ε_s). When these are specified, the surface-radiation parameter, RP, is fixed. To assess the effect of surface-radiation heat transfer contribution, calculations are performed for the following specific conditions:

$$T_w = 1000°C - 400°C, \qquad T_\infty = 100°C - 20°C$$

$$\varepsilon_w = \varepsilon_s = 1, \qquad x = 2\text{ cm}, \qquad Re_x = 1000$$

The nondimensional parameters (Ja_v, Pr_v, Pr_L, Ja_L, D, RP, and R) corresponding to the above conditions are inserted into Equations 14, 16, and 18 to yield the results given in Table 1. The quantities within the square brackets correspond to calculations neglecting surface radiation (that is, RP = 0) with the same nondimensional parametric values. Table 1 demonstrates that the importance of surface radiation is reduced in a subcooled liquid, but the effects are still significant at moderate and high wall temperature. For example, when $T_\infty = 20°C$, surface-radiation effects affect wall heat transfer predictions (corresponding to the conditions specified to obtain Table 1) only at very high wall temperatures, $T_w = 1000°C$. Under saturated flow conditions, the current analysis illustrates that a large underprediction may result in theoretical wall heat transfer predictions if surface radiation is neglected. However, the effect of the assumptions on the results may become enhanced as the bulk liquid subcooling decreases and the liquid temperature approaches the saturation temperature, as discussed below.

The current analysis is approximate in nature because the following assumptions were imposed on the current analysis: negligible effect of the vertical component of velocity of liquid (at the liquid–vapor interface) on the interfacial shear stress exerted by the liquid, negligible streamwise variation of the interfacial velocity, negligible streamwise dependence of the boundary-layer thickness ratio (ξ), and the ratio of the radiative heat transfer coefficient to the conductive heat transfer coefficient (h_r/h_c), respectively. Our comparisons of the current "moving wall analogy" approach with numerical simulations for the

Table 1 Surface-radiation effect on wall heat transfer and skin friction predictions

T_w (°C)	$\frac{Nu_x}{\sqrt{Re_x}}\left(\frac{\mu_v}{\mu_L}\right)$	$\frac{C_{fx}\sqrt{Re_x}}{2}$
Subcooled conditions (T_∞ = 20°C)		
1000	0.1039	0.00006
	[0.0539]	[0.0489]
800	0.0739	0.0055
	[0.0695]	[0.0618]
600	0.1017	0.0536
	[0.0991]	[0.0833]
400	0.1667	0.1173
	[0.1656]	[0.1247]
Subcooled conditions (T_∞ = 60°C)		
1000	0.1370	0.00006
	[0.0319]	[0.0301]
800	0.0904	0.0002
	[0.0420]	[0.0389]
600	0.0630	0.0008
	[0.0598]	[0.0537]
400	0.1024	0.0721
	[0.1009]	[0.0844]
Saturated conditions (T_∞ = 100°C)		
1000	0.1712	0.00006
	[0.0027]	[0.0027]
800	0.1127	0.0090
	[0.0032]	[0.0032]
600	0.0690	0.00022
	[0.0039]	[0.0038]
400	0.0245	0.0010
	[0.005]	[0.00497]

"no-radiation" case revealed that the present approach is very accurate in subcooled ($Ja_L = 0.15$) conditions.[9] However, in saturated flow conditions ($Ja_L = 0$), the local heat transfer parameter calculations of the moving wall approach deviate from the numerical simulations as the wall superheat parameter (Ja_v) increases towards 1. For example, under the saturated flow conditions ($Ja_L = 0$, $Pr_v = 1$, $Pr_L = 2.5$, $R = 0.21 * 10^{-4}$), the heat transfer predictions of the current "moving wall analogy" approach differ from the numerical calculations by about 15% at $Ja_v = 0.9$. This deviation was attributed to the first three assumptions. It is conjectured that with the inclusion of surface-radiation effects and because of the above assumptions, the accuracy of the "moving wall" approach may show a similar degeneracy in saturated flow conditions at "higher" wall superheats, as observed previously.[9] Nevertheless, saturation conditions are considered in this analysis because the general trends of the results may be beneficial.

An appropriate test for the current closed-form analysis would be its comparison against more accurate studies considering surface radiation. To our knowledge, unfortunately, forced-convection film boiling heat transfer analyses, which consider surface radiation along with liquid subcooling, are not available at present. As discussed earlier, the applicability of assumptions 5, 6, and 9–11 may degenerate as the liquid subcooling decreases. It is difficult to assess the competing effect of each assumption separately. With this perspective, we compared the results of the current model under the worst-case scenario (saturated liquid conditions) with previously published results (Table 2). The current model appears to fare well against the series solutions presented by Yeh and Yang.[2] A maximum overprediction of 16% ($T_w = 536$°C) is observed. This good agreement in saturated flow conditions (within the compared wall temperature range) may be due to the fact that the heat transfer characteristics of Yeh and Yang[2] are still dominated by conduction heat transfer mode and radiation is not yet dominant. Comparisons with the results of Zumbrunnen et al.[1] (where radiation is the dominant mode) in Table 2 reveal that the current results differ by more than a factor of 2 at $T_w = 1000$°C. So, the current approach may overpredict wall heat transfer coefficients at saturated flow conditions.

Table 2 Comparisons of current results with Yeh and Yang[2] and Zumbrunnen et al.[1]

(T_∞ = 100°C, x = 0.02 m)			
T_w (°C)	U_∞ (m/s)	$\left[\frac{Nu_x}{\sqrt{Re_x}}\left(\frac{\mu_v}{\mu_L}\right)\right]_{(ref. 2)}$	$\left[\frac{Nu_x}{\sqrt{Re_x}}\left(\frac{\mu_v}{\mu_L}\right)\right]_{(present)}$
536	3	0.0058	0.0067
283	3	0.0073	0.0075
144	3	0.0136	0.0137
283	1.5	0.0075	0.080
144	1.5	0.0137	0.0140

(T_∞ = 100°C, x = 0.5 m, U_∞ = 5 m/s)		
T_w (°C)	$\left[\left(\frac{h_T}{h_{Co}}\right)_{av}\right]_{(ref. 1)}$	$\left[\left(\frac{h_T}{h_{Co}}\right)\right]_{(present)}$
400	1.4	2.25
800	3.4	8.09
1000	7.1	16.85

Conclusions

A new approach that uses previously developed information on a boundary-layer flow of a liquid along a moving surface is suggested for analyzing the laminar forced-convection film boiling (with surface radiation) along a horizontal flat plate. The advantage of this approach is that the simultaneous solution of the vapor and liquid flow, which is the usual strategy, is circumvented, resulting in a simplified analysis. The "moving wall analogy" approach is most accurate under subcooled conditions, and the accuracy degenerates in saturated flow conditions at higher wall temperatures. For a water–steam system at atmospheric pressure, the theoretical wall heat transfer predictions during subcooled ($Ja_L = 0.15$) laminar forced-convection film boiling on a horizontal flat plate are affected negligibly ($T_w < 800$°C) by the exclusion of surface radiation from the analysis. As the subcooling of the liquid decreases, the general trends of the heat transfer predictions (Table 1) suggest that the surface radiation may not be a negligible quantity.

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